(3.5) is replaced by D and $\alpha = -e_{11}/\epsilon_{11}^s$[11]. For most materials dR/dD will be negligible. Moreover, for weak shocks $\rho_0 R\, du/dp \simeq 1$; then if $M\partial p_x/\partial x$ is negligible, as is often the case, equation (3.16) becomes

$$\frac{Dp_x}{Dt} = -\frac{e_{11}/\epsilon_{11}^s}{1 + a^2/(R-u)^2}\frac{dD}{dt} \simeq -\frac{e_{11}}{2\epsilon_{11}^s}\frac{dD}{dt}.$$

For the kind of device assumed, current in the external circuit, $I(t)$, is proportional to dD/dt, so

$$\frac{Dp_x}{Dt} \simeq -e_{11}I(t)/2C\epsilon_{11}^s \tag{3.20}$$

where C is cross-section area of the shocked material[12]. If the external load is resistive, $I(t) > 0$ and the analogy with the relaxing solid is close. If the external load has a resonance, $I(t)$ may alternate signs, causing an analogous oscillation in the decay rate.

(iii) *Viscous fluids.* For a Newtonian fluid with shear viscosity ν,

$$p_x = p(v) + 4\nu\dot{\epsilon}_x/3 \tag{3.21}$$

where $\xi = \dot{\epsilon}_x$ and $\alpha = 4\nu/3$. Equation (3.5) becomes

$$\dot{p} = \rho c^2 \dot{\epsilon}_x + 4\nu\ddot{\epsilon}_x/3 \tag{3.22}$$

where c^2 is the hydrodynamic sound speed and $\dot{\epsilon}_x = (1/\rho)\, d\rho/dt$.

This is an interesting case because the form of equation (3.21) precludes the possibility of a discontinuous shock front. Were this to occur, $\dot{\epsilon}_x$, and therefore p_x, would become infinite, which is a contradiction. The jump conditions apply also to a steady transition connecting two uniform states, but that case is not interesting in the present context. If the shock transition is followed by a rarefaction, the rarefaction and shock interfere in some degree, so the jump conditions are no longer exact. This is apparent from equation (3.22), which shows that p and ρ do not simultaneously achieve their maximum values. The locus of states connecting shock and rarefaction does not, therefore, form a cusp. Instead, it is a continuous convex curve connecting the shock line and the expansion isentrope. Moreover, the entire problem of defining a propagation velocity is reopened and cannot readily be settled in a satisfactory way[24]. However, numerical calculations have shown that under such conditions as will exist, the lower part of the shock profile will still closely conform to the steady profile. Also the shock jump conditions still describe the relations among propagation velocity, peak pressure, particle velocity and density reasonably well. In other words, the locus of p, v states through even a non-steady shock will not deviate very much from the Rayleigh line, except for very large viscosities. The pressure profile in this unsteady quasi-shock will be somewhat as shown in Fig. 2. If the shock jump conditions are used to connect state A and the initial state, equation (3.16) applies. With $B = 0$ and the approximations $R - u \simeq c$ and $dp_x/du = R$, it becomes

$$\frac{Dp_x}{Dt} \simeq (R - u - c)\frac{\partial p_x}{\partial x} + \frac{2\nu}{3}\ddot{\epsilon}_x. \tag{3.23}$$

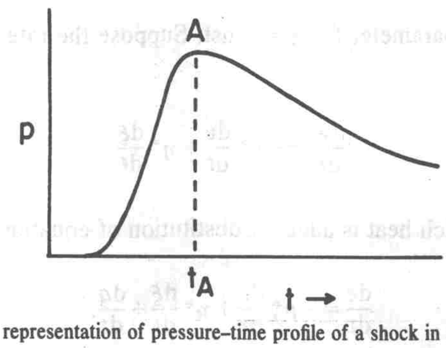

Fig. 2. Schematic representation of pressure–time profile of a shock in a viscous medium.

Using equation (3.7), equation (3.23) becomes

$$\frac{\mathrm{D}p_x}{\mathrm{D}t} \simeq \left(1 + \frac{u-R}{c}\right)\dot{p}_x + \frac{2\nu}{3}\ddot{\epsilon}_x. \tag{3.24}$$

At A, $\dot{p}_x = 0$, and from equation (3.22) $2\nu\ddot{\epsilon}_x/3 = -\rho c^2 \dot{\epsilon}_x{}^A/2$. That is, the rate of decay of the shock depends on strain rate at A, and this in turn depends directly on the viscosity, ν. This result is in marked contrast with that of equations (2.1) and (2.2) and is not easily explicable.

If the peak of the $\rho(t)$ profile is chosen, instead of $p_x(t)$, the situation is altered. Now $\dot{\epsilon} = 0$ and $\dot{p}_x = 4\nu\ddot{\epsilon}_x{}^A/3$ and equation (3.24) becomes

$$\begin{aligned}\frac{\mathrm{D}p_x}{\mathrm{D}t} &\simeq \left(\frac{3}{2} + \frac{u-R}{c}\right)\left(\frac{\mathrm{d}p_x}{\mathrm{d}t}\right)_{\dot{\epsilon}_x=0} \\ &= \left(\frac{3}{2} + \frac{u-R}{c}\right)\cdot\frac{4\nu}{3}(\ddot{\epsilon}_x)_{\dot{\epsilon}_x=0} \\ &\simeq 2\nu(\ddot{\epsilon}_x)_{\dot{\epsilon}_x=0}.\end{aligned} \tag{3.25}$$

Since characteristics in the rarefaction following the shock are still expected to overtake the shock and to provide something approximating the usual hydrodynamic attenuation, it seems likely that the viscosity term produces an additional attenuation. Whether or not this additional attenuation is significant remains to be determined.

(iv) *Plastic shear-yielding solids.* If, in this case, a shear stress, τ, proportional to plastic strain rate exists and there is no plastic dilatation,

$$\tau = 2\nu(\dot{\epsilon}_x{}^p - \dot{\epsilon}_y{}^p)/2 \qquad p_x = p_S(v) + 2\nu\dot{\epsilon}_x{}^p$$

where $p_S(v)$ represents the quasi-static dependence of p on v. Now $\xi = \epsilon_x{}^p$, $\alpha = 2\nu$, and $B = 0$. The problem is exactly analogous to that for a viscous fluid.

Combinations of the above effects are possible. Extension of the analysis to include two or more variables, ξ_1, ξ_2, etc. is straightforward. Similar relations have been used to describe wave propagation in chemically-reacting media[13, 14]. Coleman, Chen and others have obtained similar equations for a variety of problems[15–18].

4. THERMAL EFFECTS

Suppose that flow behind the shock is entropic and that $\xi = S$, the specific entropy. Equation (3.16) follows, as before, but it now contains derivatives of S. These can be eliminated through use of equation (3.3) and some assumptions about the underlying thermodynamics of the material.

Let

$$p_x^* = p_x^*(v, S, \xi).$$

Then

$$\frac{\mathrm{d}p_x^*}{\mathrm{d}t} = a^{*2}\frac{\mathrm{d}\rho}{\mathrm{d}t} + \frac{\Gamma^* T}{v}\frac{\mathrm{d}S}{\mathrm{d}t} + a^*\frac{\mathrm{d}\xi}{\mathrm{d}t} \tag{4.1}$$

where Γ is the Grüneisen parameter for $\xi = \text{const}$. Suppose the rate at which work is done on unit mass is

$$\frac{\mathrm{d}w}{\mathrm{d}t} = -p_x^*\frac{\mathrm{d}v}{\mathrm{d}t} + \eta^*\frac{\mathrm{d}\xi}{\mathrm{d}t} \tag{4.2}$$

and $\mathrm{d}q/\mathrm{d}t$ is the rate at which heat is added. Substitution of equation (4.2) into (3.3) gives

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -p_x^*\frac{\mathrm{d}v}{\mathrm{d}t} + \eta^*\frac{\mathrm{d}\xi}{\mathrm{d}t} + \frac{\mathrm{d}q}{\mathrm{d}t}. \tag{4.3}$$